Power System Transient Stability Simulation

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Overview

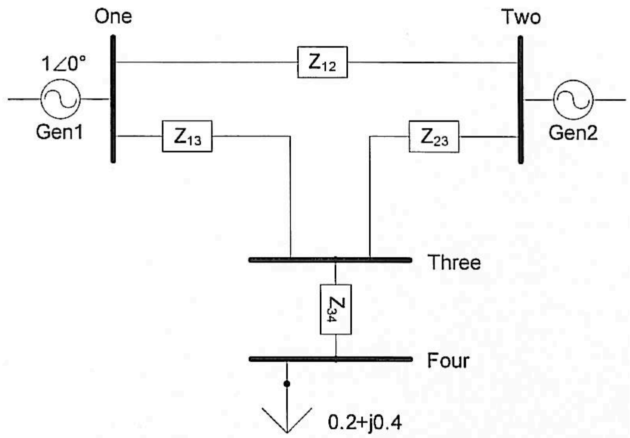
Electricity is an intrinsic part of modern culture, and having reliable access is a necessity. Power supply systems, i.e. power grids, have the responsibility to produce and distribute steady power to all consumers. Due to the nature of power, and of alternating currents, slight disturbances can cause large amounts of fluctuation and damage quickly. These can cause power outages that can range in severity from a single downed line to an entire country or more without power.

The purpose of this project is to create accurate simulations of power outages that can be used to avoid actual power failures. For the simulations to be useful, they must be able to run faster than real time, to determine what will happen when there's an outage before the outcome occurs. To accomplish this, the Parareal in Time Algorithm is being implemented to increase speed up and aid in a logical way to parallelize the code.

Steady State System

To simulate power outages, one must first be able to simulate and model a steady state power system that has no issues. A simulation is accurate when the amount of power generated is very closely matched to the amount of power used, and all voltages and voltage angles are known. This starts with the physics of electricity, magnetism, and circuits, and the design of power grids. An accurate representation of a power grid includes all the lines, buses, transformers, generators, loads, and many other pieces. Each line has an impedance (z) associated with it, where impedance is the total opposition to the alternating current (Dictionary.com).

Admittances (y) are more commonly used in calculations, and are the inverses of each impedance value. The admittances for each line are put into an admittance matrix (Y) with the diagonal entries being all values flowing into a bus, and the rest being the value between two buses, with unconnected buses having a value of zero.



A model of a two generator and four bus power system. [5]

The buses are broken up into three categories based on the given information for each. Load buses, or PQ buses, have their real and imaginary power specified. Generator buses, or PV buses, have their real power and their voltage magnitude specified. One bus, the slack bus, has the voltage angle and voltage magnitude specified [4].

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$Y = \begin{bmatrix} (y_{11} + y_{13}) & -y_{12} & -y_{13} & 0 \\ -y_{12} & (y_{12} + y_{23}) & -y_{23} & 0 \\ -y_{13} & -y_{23} & (y_{13} + y_{23} + y_{34}) & -y_{34} \\ 0 & 0 & -y_{34} & y_{34} \end{bmatrix}$$

Admittance Matrices for the two generator four bus system [5]

Due to the nature of alternating currents, the admittances are complex numbers, with the real part being the conductance (g) and the imaginary part being the susceptance (b). Because the admittances are complex, so too is the power.

The real and imaginary power is calculated as follows [4]:

$$P_i^{sp} = P_i(\theta, V) = V_i \sum_{k=1}^{n} V_k(G_{ik}\, cos\theta_{ik} + B_{ik} sin\theta_{ik})$$

$$Q_i^{sp} = Q_i(\theta, V) = V_i \sum_{k=1}^{n} V_k(G_{ik}\, sin\theta_{ik} + B_{ik} cos\theta_{ik})$$

Each PV bus will have a real power equation (P), and each PQ bus will have both a real power equation (P) and an imaginary power equation (Q). These equations will be used to solve for the voltage and voltage angles. The equations will be coupled because the voltage at one bus is influenced by the voltage and power elsewhere.

Due to the coupling, the equations must be solved numerically rather than exactly. The method used to solve this system of equations in Newton's Method. Since the power is known for all of the buses except the slack bus, those values are set against the equations listed above, and the equations are solved iteratively until the two sides agree or a certain margin of error is reached.

Initial values for the states are the first things that need to be decided. It's a common practice to use a flat start, meaning that all of the buses have a voltage of one per unit and a voltage angle of zero [5]. A method is then used to correct the values, based on the amount of error, and a second iteration is run.

Dynamic System

The final solution values for the steady state functions are used as the initial state values of the system for the dynamic problem. They represent the state of the power grid before there is a fault. To accurately simulate the fault and ensuing changes to the system, time dependent functions for many of the state variables are used, as well as a few algebraic equations. The equations are as follows, where $w_B, H, D, T'_{do}, X_d, X_d', T_{qo}'$, $X_q, X_q', T_c, T_{sv}, R_d, T_E, K_E, A_E, e^{(B_E E_{fd})}, K_F, T_F, T_{CH}, R_a, B_{ij}$, and G_{ij} are constants [2].

Name	Equation	MATLAB Function
Stator Algebraic Equation	$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \frac{1}{(R_a^2 + \acute{X}_d \acute{X}_q)} \begin{bmatrix} R_a & \acute{X}_d \\ -\acute{X}_q & R_a \end{bmatrix} \begin{bmatrix} \acute{E}_q - V_q \\ \acute{E}_d - V_q \end{bmatrix}$	Eq_StatorAlgebraic22.m
Network Algebraic Equations	$Y_{ij}^{DQ} = \begin{bmatrix} B_{ij} & G_{ij} \\ G_{ij} & -B_{ij} \end{bmatrix};\ V_j^{DQ} = \begin{bmatrix} \frac{V_{Qj}}{V_{Dj}} \end{bmatrix};\ I_i^{DQ} = \begin{bmatrix} \frac{I_{Di}}{I_{Qi}} \end{bmatrix};$	NWAlgebraic22.m
Governor Model	$\frac{dP_{sv}}{dt} = \frac{1}{T_{sv}} \left[-P_{sv} + P_c - \frac{1}{R_D} S_m\right]$	Eq_SteamGov.m
Turbine Model	$\frac{dT_m}{dt} = \frac{1}{T_{CH}} [-T_m + P_{SV}]$	Eq_SteamTurb.m
Change in q - axis Transient Voltage	$\frac{dE'_q}{dt} = \frac{1}{T'_{do}} \left[-E'_q + (X_d - X'_d)I_d + E_{fd}\right]$	Eq_ExcType1.m
Change in d-axis Transient Voltage	$\frac{dE'_d}{dt} = \frac{1}{T'_{qo}} \left[-E'_d - \left(X_q - X'_q\right)I_q\right]$	Eq_ExcType1.m
Change in Exciter Field Voltage	$\frac{dE_{fd}}{dt} = \frac{1}{T_E} \left[-\left(K_E + A_E \left(e^{(B_E E_{fd})}\right)\right) E_{fd} + V_R\right]$	Eq_ExcType1.m
Change in Rotor Angle	$\frac{d\delta}{dt} = W_B S_m$	Eq_Gen22.m
Change in Slip	$\frac{dS_m}{dt} = \frac{1}{2H} [-DS_m + T_m - T_e]$	Eq_Gen22.m
Change in DC Voltage	$\frac{dE_{dc}}{dt} = \frac{1}{T_c} \left[-E_{dc} - \left(X'_q - X'_d\right)I_q\right]$	Eq_ExcType1.m

$$\frac{d\vec{x}}{dt} = \vec{f}(t, \vec{x})$$

$$\vec{x}_{n+1} = \vec{x}_n + \frac{1}{6}\left(\vec{k}_1 + 2\vec{k}_2 + 2\vec{k}_3 + \vec{k}_4\right)$$

$$\vec{k}_1 = h \cdot \vec{f}(t_n, \vec{x}_n)$$

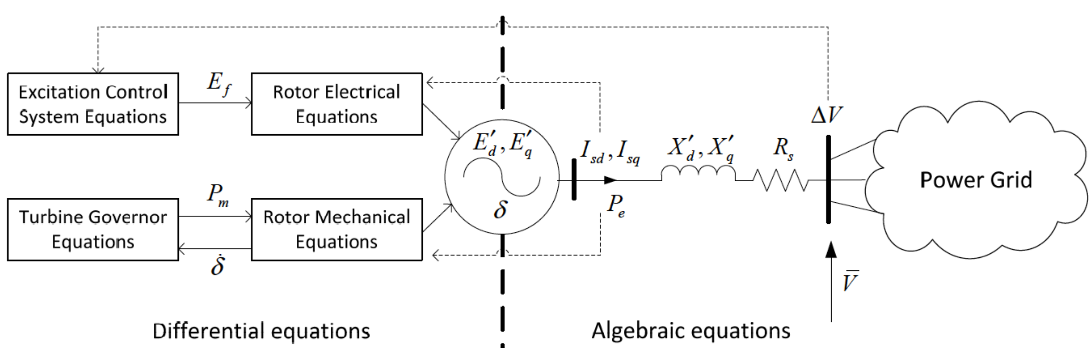
$$\vec{k}_2 = h \cdot \vec{f}\left(t_n + \frac{1}{2}h,\ \vec{x}_n + \frac{1}{2}\vec{k}_1\right)$$

$$\vec{k}_3 = h \cdot \vec{f}\left(t_n + \frac{1}{2}h,\ \vec{x}_n + \frac{1}{2}\vec{k}_2\right)$$

$$\vec{k}_4 = h \cdot \vec{f}\left(t_n + h,\ \vec{x}_n + \vec{k}_3\right)$$

The Runge-Kutta 4 Method

The Runge-Kutta 4 method is used to solve for the values of the state variables at each time step. K1, K2, K3, and K4 are calculated for each of the MATLAB functions listed above and h is the time step used in the calculations.

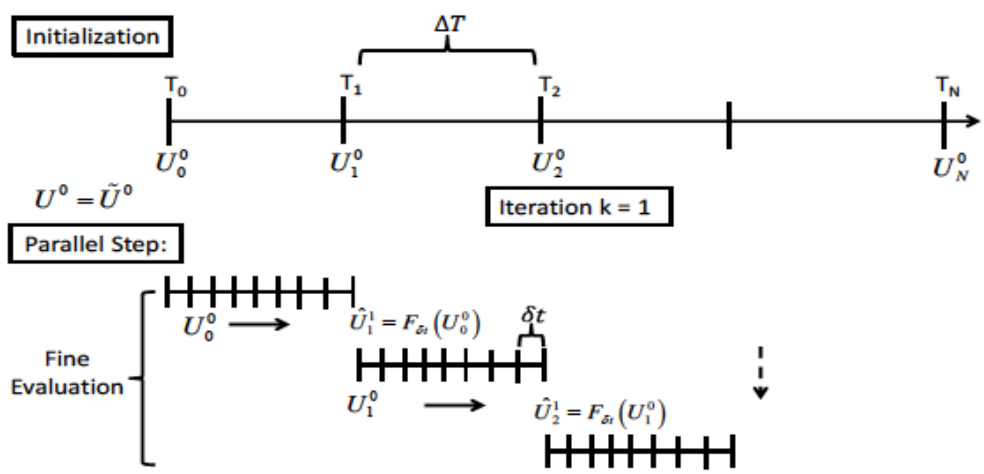


Visual Representation of how equations relate to the physical grid [2]

Parareal In Time Algorithm

To make the calculations easier to manage and faster to solve, they are broken up into sections. Because of how interconnected the system is, it is not possible to use spatial decomposition. Instead, a method known as the Parareal in Time Algorithm is used.

The Parareal in Time algorithm breaks the system up within the time domain, rather than the physical domain [3]. The time axis is broken into sections, and the calculations that would have been done along the entire axis are instead done on each smaller section. The results for the small sections are then attached together in the order designated by their location within the original axis.



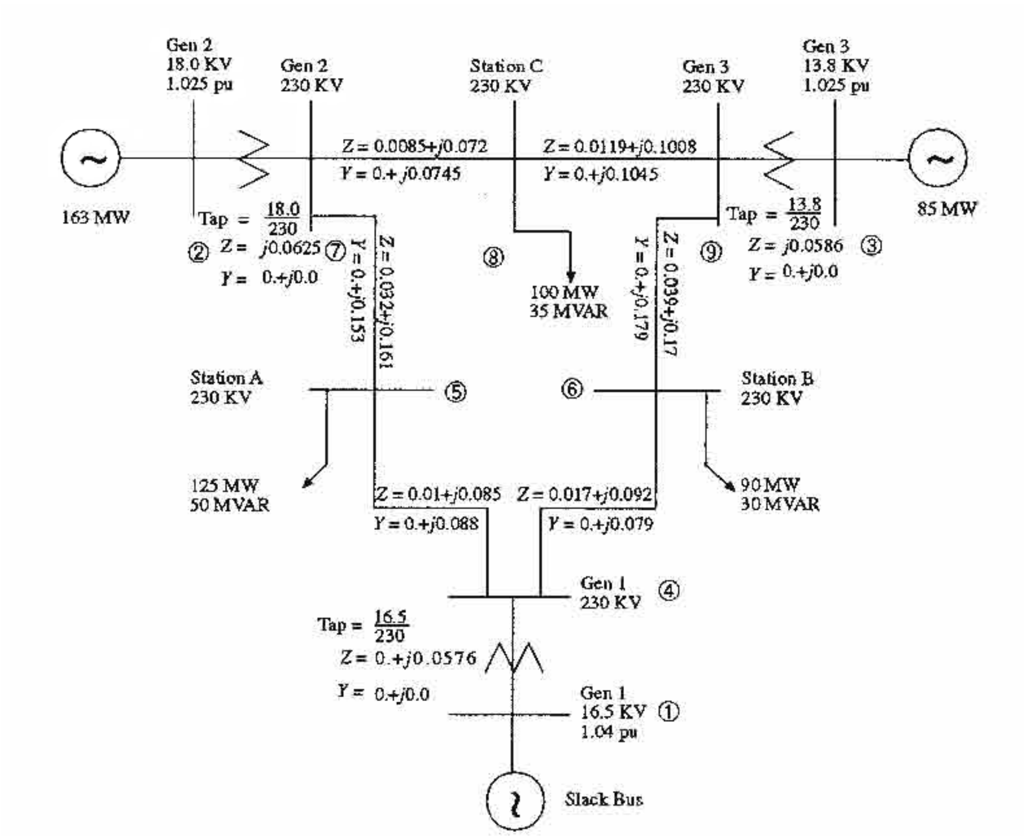
Parallel Implementation of the Parareal Design for Dynamic Power Flow Analysis [3]

The biggest complication to overcome when using this algorithm is finding a starting value for each section. To solve a differential equation, a function and a beginning value are both needed, which is covered for the main axis by the values given by the steady state system. The steady state system only gives values for time zero though, and no initial values are known for all but the first small time section.

To get the starting values for each sub-section, a coarse approximation function is used on the entire axis and the values at the beginning of each section are used as the starting value. The approximation function, the trapezoidal rule, is not nearly as accurate as the RK4 function, so the values calculated based on those starting values will not be as close to the correct values as the RK4 method would be, but will be good enough to use as a starting point. Once a fine solve iteration has run, the coarse value is corrected, the coarse method is reran with a more accurate value and the system runs again. This iterative method is used until the value is within a specified margin of error, or the maximum number of iterations is reached.

To make this process run faster than simply solving each time step one after the other, the calculations for each of the sub-intervals are done in parallel. When the number of sub-intervals is large, doing the calculations in parallel can provide a significant speed up. The speed up is offset by the time it takes for the coarse solver and coarse corrector to run, which a simple serial calculation would not have.

3 Generator 9 Bus System



Power model of a three generator and nine bus system with power amounts, impedance values, and admittance values [2]

The starting values for the 3 generator 9 bus system are as shown in the above diagram. The steady state system is solved using MatPower (matpower5.1) [7], a module added into the MATLAB code framework. Results in Appendix A. All results have been found using this system.

Results

To test the parallel capabilities of MATLAB, two random nxn matrices were generated and multiplied together within a for loop, as well as within a parfor loop, which does the calculations in parallel rather than sequentially. The time was measured for different matrix sizes as well as the number of iterations. The results of some of the tests are in the following table.

Matrix Size	Number of Iterations	Serial Time	Parallel Time
10,000	32	84.986s	119.811s
10,000	64	267.242s	232.232s
5,000	1024	401s	314s
5,000	512	236s	262s

The table shows that once the number of iterations is large, or the matrix size is large and the number of iterations is relatively large, the time for the parallel code to run is shorter than the time for the serial code to run. This leads to the conclusion that MATLAB has a high time cost for setting up parallel computations. This makes using the parfor loop useful only for large systems where the setup cost will be minor compared to the time that the loop will run.

For the three generator and nine bus system, the section of code that iterates over the coarse sections to solve the RK4 function uses a regular for loop for the serial code, the fnParaReal22.m file. Psuedocode is displayed in the box below. To implement the Parareal in Time Algorithm correctly, the for loop was replaced with a parfor loop, and many of the data structures were rearranged to meet the requirements for running in parallel (done by Aleksandar Dimitrovski).

Trapezoid Function Call – Initial coarse evaluation

While iterations less than max number of iterations:

For each coarse section (in parallel):

Runge-Kutta 4 Function Calls - fine evaluation

Correct coarse evaluation

Add one to iteration count

Psuedocode of the fnParaReal22.m loop

The parallel code was run on thirty-two workers on the nanoheat1 machine from the University of Tennessee. The theoretical speed up was calculated based on the speed up algorithm in "Parareal in Time for Fast Power System Dynamic Simulations" [3], which is N/k where N is the number of intervals and k is the number of iterations for all intervals.

The theoretical speed up was calculated to be approximately 5.33 for the Fige3g9b_C1_BusF_1_6c file, which used six iterations. This case was chosen because it required the most iterations to converge, and would give a "worst case scenario" for the theoretical speed up. The value used for N was 32, because there were only 32 workers and the code could not run more than 32 intervals at a time regardless of the fact that the time interval was actually broken into 450 segments.

The time for the serial loop to complete took 5.423 seconds. The time for the parallel loop to complete took 1.151 seconds. Tables for both the serial and parallel loop times are located in appendix B. The measured speed up was then calculated by dividing the serial loop time by the parallel loop time. This came out to be approximately 4.7.

Conclusion and Future Work

Based on the results above, we were able to conclude that the Parareal in Time algorithm within MATLAB will give a speed increase to running the simulation. The next steps will be to calculate speed ups for more the of the fault cases for the three generator and nine bus system and then run the simulation for large systems and see how the speed up scales with the system. Once it has been determined that the speed up is consistent across multiple systems, C, C++, or Fortran code should be written from scratch. The new code would be more flexible with parallel implementation and code optimization.

Appendix A

[7]

MATPOWER Version 5.1, 20-Mar-2015 -- AC Power Flow (Newton)
Newton's method power flow converged in 4 iterations.
Converged in 0.13 seconds

System Summary

How Many?		How Much?	P (MW)	Q (Mvar)
Buses	10	Total Gen Capacity	820.0	-900.00 to 900.00
Generators	3	On-line Capacity	820.0	-900.00 to 900.00
Committed Gens	3	Generation (actual)	319.6	22.8
Loads	3	Load	315.0	115.0
Fixed	3	Fixed	315.0	115.0
Dispatchable	0	Dispatchable	-0.0 of -0.0	-0.0
Shunts	0	Shunt (inj)	-0.0	0.0
Branches	11	Losses (I^2 * Z)	4.64	48.38
Transformers	0	Branch Charging (inj)	-	140.5
Inter-ties	0	Total Inter-tie Flow	0.0	0.0
Areas	1			

	Minimum	Maximum
Voltage Magnitude	0.996 p.u. @ bus 5	1.040 p.u. @ bus 1
Voltage Angle	-3.99 deg @ bus 5	9.28 deg @ bus 2
P Losses (I^2*R)	-	2.30 MW @ line 5-7
Q Losses (I^2*X)	-	15.83 Mvar @ line 2-7

Bus Data

	Voltage		Generation		Load	
Bus #	Mag (pu)	Ang (deg)	P (MW)	Q (MVAr)	P (MW)	Q (MVAr)
1	1.040	0.000	71.64	27.05	-	-
2	1.025	9.280	163.00	6.65	-	-
3	1.025	4.665	85.00	-10.86	-	-
4	1.026	-2.217	-	-	-	-
5	0.996	-3.989	-	-	125.00	50.00
6	1.013	-3.687	-	-	90.00	30.00
7	1.026	3.720	-	-	-	-
8	1.016	0.728	-	-	100.00	35.00
9	1.032	1.967	-	-	-	-
10	1.024	6.499	-	-	-	-

Total:	319.64	22.84	315.00	115.00

Branch Data

Branch #	From Bus	To Bus	From Bus Injection P (MW)	From Bus Injection Q (MVAr)	To Bus Injection P (MW)	To Bus Injection Q (MVAr)	Loss (I^2 * Z) P (MW)	Loss (I^2 * Z) Q (MVAr)
1	1	4	71.64	27.05	-71.64	-23.92	0.000	3.12
2	4	6	30.7	1.03	-30.54	-16.54	0.166	0.90
3	6	9	-59.46	-13.46	60.82	-18.07	1.354	5.90
4	3	9	85	-10.86	-85.00	14.96	0.000	4.10
5	8	9	-24.1	-24.30	24.18	3.12	0.088	0.75
6	7	8	76.38	-0.8	-75.90	-10.70	0.475	4.03
7	2	7	163	6.65	-163.00	9.18	0.000	15.83
8	5	7	-84.32	-11.31	86.62	-8.38	2.300	11.57
9	4	5	40.94	22.89	-40.68	-38.69	0.258	2.19
10	10	2	-0.00	0.00	0.00	-0.00	0.000	0.00
11	10	7	0.00	-0.00	-0.00	0.00	0.000	0.00
						Total:	4.641	48.38

```
Vg0bar =
  1.0400 + 0.0000i
  1.0116 + 0.1653i
  1.0216 + 0.0834i
```

Appendix B

0.317865
0.173421
0.155791
0.157043
0.154127
0.193035

Parallel Loop Times

0.96587
0.943508
0.907619
0.891114
0.866328
0.848641

Serial Loop Times

Sources

[1] *Dictionary.com*. Dictionary.com. Web. 6 Aug. 2015.

[2] Gurrala, Gurunath. "Power System Parallel Dynamic Simulation Framework for Real Time Wide-Area Protection and Control."

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[7] Zimmerman, Ray D., and Carlos E. Murillo-Sanchez. *MatPower*. Computer software. *MatPower*. Vers. 5.1. N.p., 20 Apr. 2015. Web. <http://www.pserc.cornell.edu//matpower/>.