Vascular Fluid Structure Simulation

Au Yeung, Tak Shing, Ivan (CUHK)
Mentor: Kwai L. Wong (UTK/ORNL)

Overview

This project is to simulate vascular flow in arteries. The vascular system is an important component for the human health and a computational model of blood flow could help diagnosis and treatment of health problems.

Also, this project evaluates the stability of the solver to handle fluid structure interaction problem with the boundary implementation.

Blood flow is described by 3D cylindrical incompressible Navier-Stokes equations(INS)[1], and a set of structure equations [2] determines the radial and longitudinal deformation of the vessel wall.

Formulations

- Fluid Equations(INS)

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \upsilon\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2} - \frac{u^2}{r^2}\right)$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial x^2}\right)$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial x}$$

- Boundary Conditions

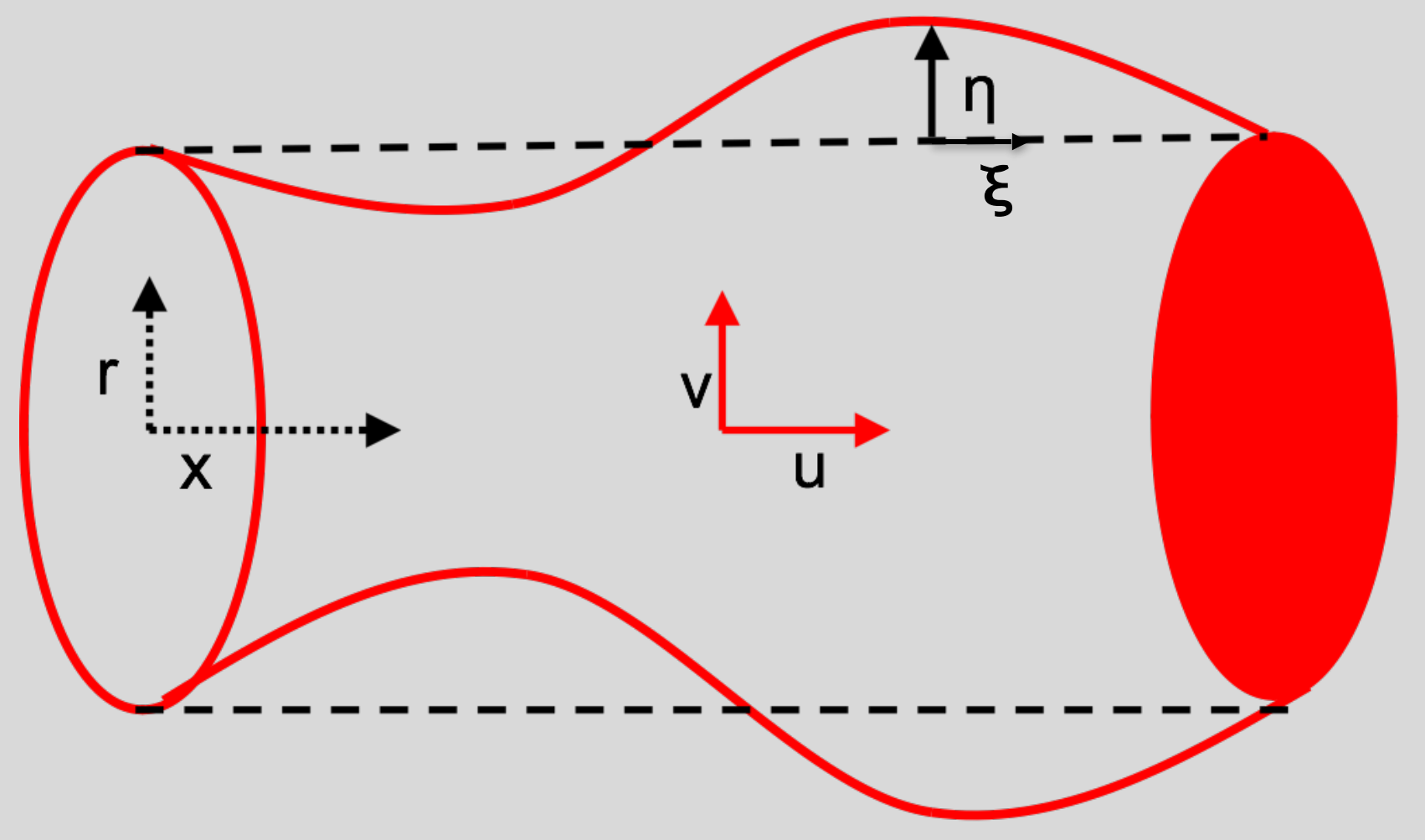
$$[u]_{r=a} = \frac{\partial \eta}{\partial t} \quad \text{and} \quad [w]_{r=a} = \frac{\partial \xi}{\partial t}$$

- Vessel Wall Equations(Structure Equations)

$$M_0\frac{\partial^2 \xi}{\partial t^2} + L_x\frac{\partial \xi}{\partial t} + K_x\xi$$
$$= \frac{E_x h}{1-\sigma_\theta\sigma_x}\frac{\partial^2 \xi}{\partial x^2} + \left(\frac{T_{t_0} - T_{\theta_0}}{a} - \frac{E_x h\sigma_x}{a(1-\sigma_\theta\sigma_x)}\right)\frac{\partial \eta}{\partial x} - \mu\left[\frac{\partial w}{\partial r} + \frac{\partial u}{\partial x}\right]_a$$

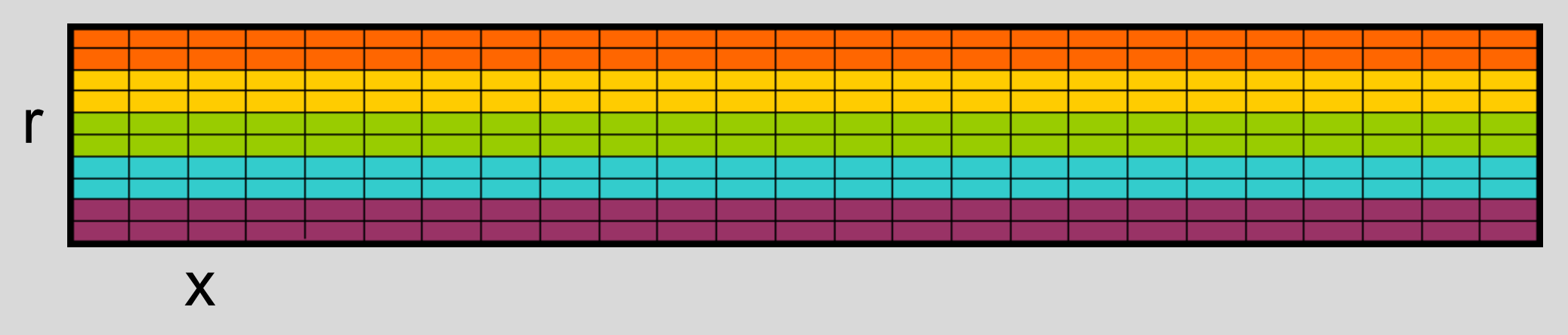
$$M_0\frac{\partial^2 \eta}{\partial t^2} + L_r\frac{\partial \eta}{\partial t} + K_r\eta$$
$$= T_{t_0}\frac{\partial^2 \eta}{\partial x^2} + \left(\frac{T_{\theta_0}}{a^2} - \frac{E_\theta h}{a^2(1-\sigma_\theta\sigma_x)}\right)\eta + \frac{E_\theta h\sigma_\theta}{a(1-\sigma_\theta\sigma_x)}\frac{\partial \xi}{\partial x} + \left[p - 2\mu\frac{\partial u}{\partial r}\right]_a$$

Artery model



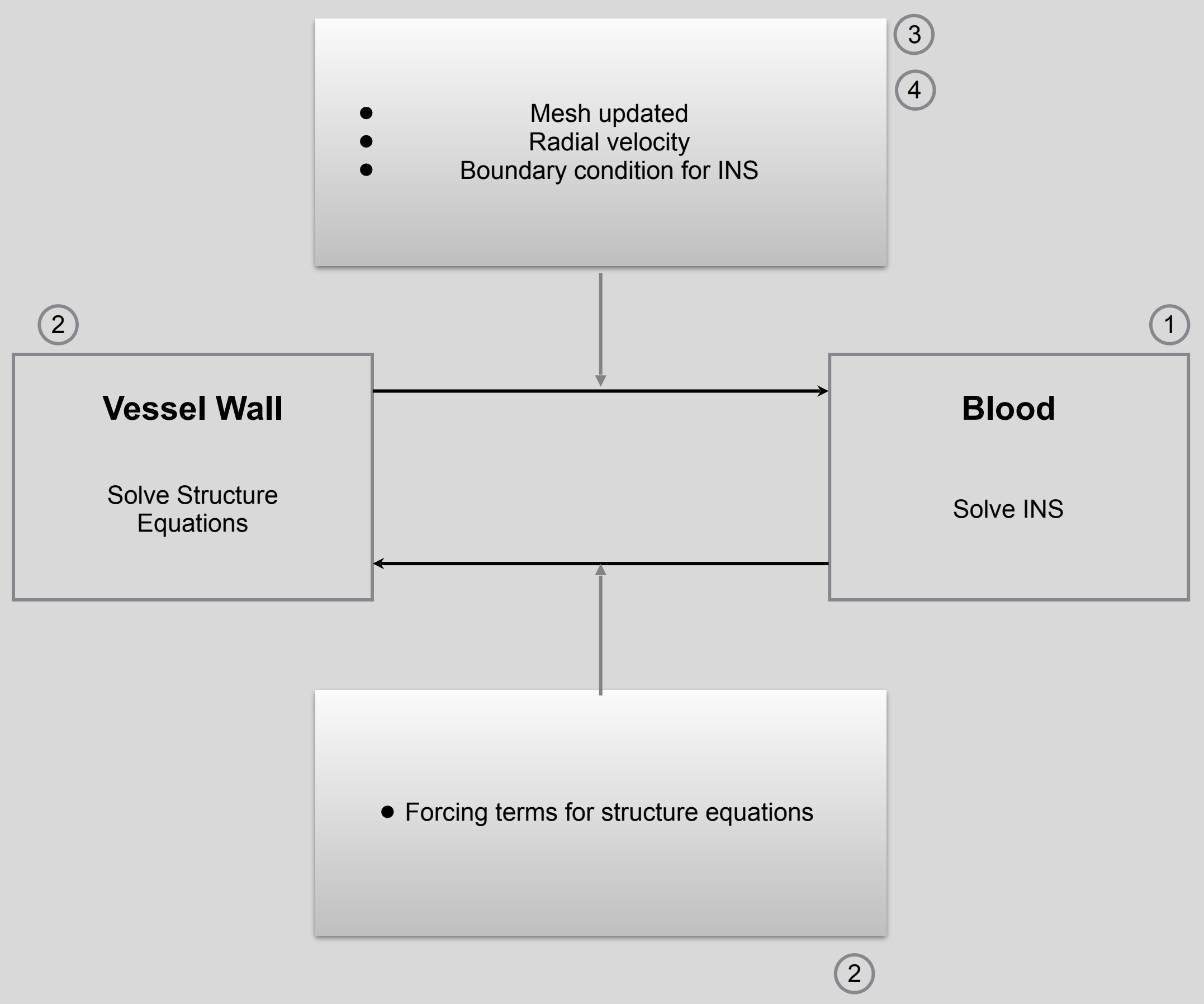
Parallel Computation

- PICMSS
- 5 processors
- Each responsible for several rows of grid
- 1cm diameter x 6cm length



Algorithm

1. Solve Navier-Stokes equations(INS) for blood velocity(u,w) and pressure(p) on a 2D mesh
2. Solve structure equations for radial and longitudinal deformations(η,ξ) of vessel wall on a 1D mesh
3. Update mesh using η , ξ , since vessel wall has moved
4. Update radial velocity at vessel wall, since radial blood velocity at vessel wall must equal radial wall velocity
5. Repeat Step 1-4 until a stable solution is reached
6. t = t + Δt
7. Continue from Step 1



Methodology

To solve INS:

Parallel Interoperable Computational Mechanics System Simulator(PICMSS) was chosen to solve INS.

- PICMSS

PICMSS is a parallel computational software for solving equations with continuous Galerkin finite element method developed by Dr. Kwai Wong, University of Tennessee. PICMSS is written in C program with MPI and uses Trilinos iterative library for solving systems of linear equations generated internally by finite element method. The finite element library includes 2D triangle and quadrilateral, and 3D tetrahedron and hexahedron master elements. PICMSS is capable of admitting various formulations of fluid flow problems directly written in partial differential equation(PDE) template operator form.

- Finite Element Method

This method divides the domain into parts and over each parts, uses some element functions to seek approximate solution then assembles the parts.

To solve Structure Equations :

1. Use continuous Galerkin finite element method
2. Use Newmark method to solve system of second order PDE

-Newmark Method

This method involves equations of the form:

$$[M]\{\frac{\partial^2\eta}{\partial t^2}\} + [C]\{\frac{\partial\eta}{\partial t}\} + [K]\{\eta\} = F$$

The solution of this equation for the Newmark Method is :

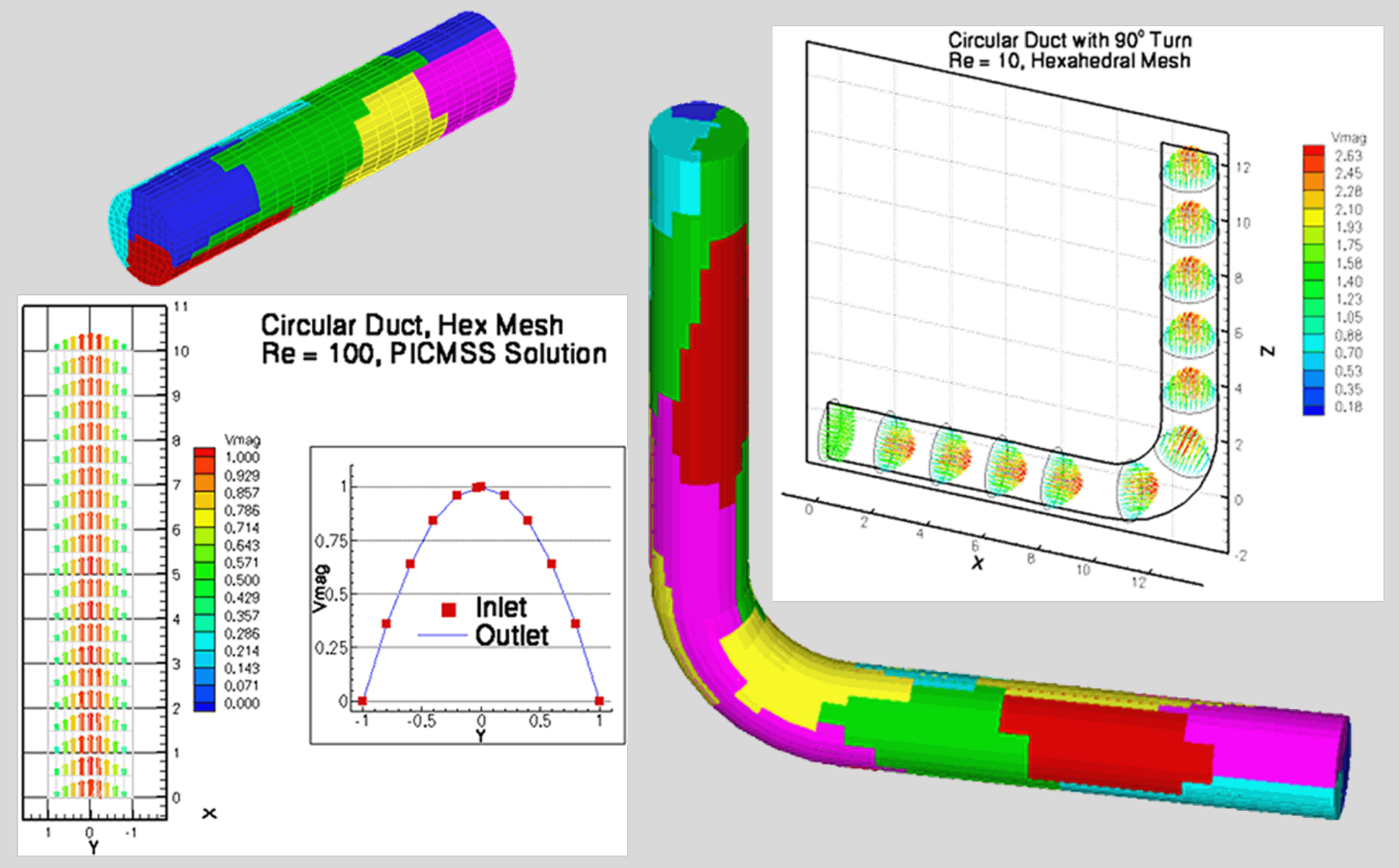
$$([M] + \frac{\delta t}{2}[C] + \frac{\delta t^2}{4}[K])\{\frac{\partial^2\eta}{\partial t^2}\}_{n+1}$$
$$= [F]_{n+1} - [C](\{\frac{\partial\eta}{\partial t}\}_n + \frac{\delta}{2}\{\frac{\partial^2\eta}{\partial t^2}\}_n) - [K](\{\eta\}_n + \delta t\{\frac{\partial\eta}{\partial t}\}_n + \frac{\delta t^2}{4}\{\frac{\partial^2\eta}{\partial t^2}\}_n)$$
$$\{\eta\}_{n+1} = \{\eta\}_n + \delta t\{\frac{\partial\eta}{\partial t}\}|_n + \frac{\delta t^2}{4}(\{\frac{\partial^2\eta}{\partial t^2}\}_n + \{\frac{\partial^2\eta}{\partial t^2}\}_{n+1})$$
$$\{\frac{\partial\eta}{\partial t}\}_{n+1} = \{\frac{\partial\eta}{\partial t}\}_n + \frac{\delta t}{2}(\{\frac{\partial^2\eta}{\partial t^2}\}_n + \{\frac{\partial^2\eta}{\partial t^2}\}_{n+1})$$

Current Work

- Use PICMSS to solve INS
- Use a serial code to solve structure equations
- Use these solutions to simulate the boold flow

Future Work

- Implement 3D small artery model
- extend to 2D structure equations to simulate the vessel wall
- Use PICMSS to solve both equations and apply it to arbitrary vascular geometry



References

[1] A. Quarteroni, M. Tuveri, A. Veneziani,``Computational vascular fluid dynamics: problems, models, and methods'',Comput Visual Sci, vol. 2, pp. 163-197, 2000.

[2] J. T. Ottesen, M. S. Olufsen, J. K. Larsen, Applied Mathematical Models in Human Physiology(Siam Monographs on Mathematical Modeling and Computation), SIAM, 2004.

[3] T. Hughes, The Finite Element Method: Linear Static and Dynamic Finite Element Analysis, Mineola, New York: Dover Publications, Inc., 2000.

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