Vascular Fluid Structure Simulation

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Overview

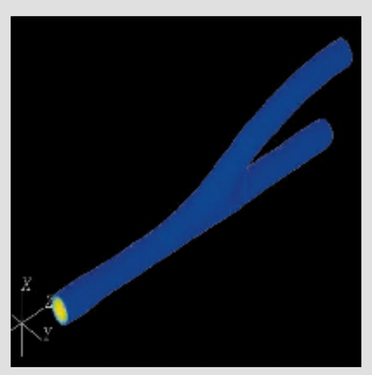
This project is to simulate vascular flow in arteries. The vascular system is an important component for the human health and a computational model of blood flow could help diagnosis and treatment of health problems.

- Evaluates the stability of implemented solvers to handle fluid structure interaction problems
- Use continuous Galerkin finite element method
- Utilize DIEL to solve weak coupling equations

Fluid Structure Interaction

- Fluid Equations(INS)

$$\begin{cases} u_t - \frac{1}{\mathrm{Re}}\nabla^2 u + u\cdot\nabla u + \nabla p = 0 \\ \nabla\cdot u = 0 \end{cases}$$



Quarteroni et al. [1]

- Boundary Conditions

$$[u]_{r=a} = \frac{\partial\eta}{\partial t} \quad \text{and} \quad [w]_{r=a} = \frac{\partial\xi}{\partial t}$$

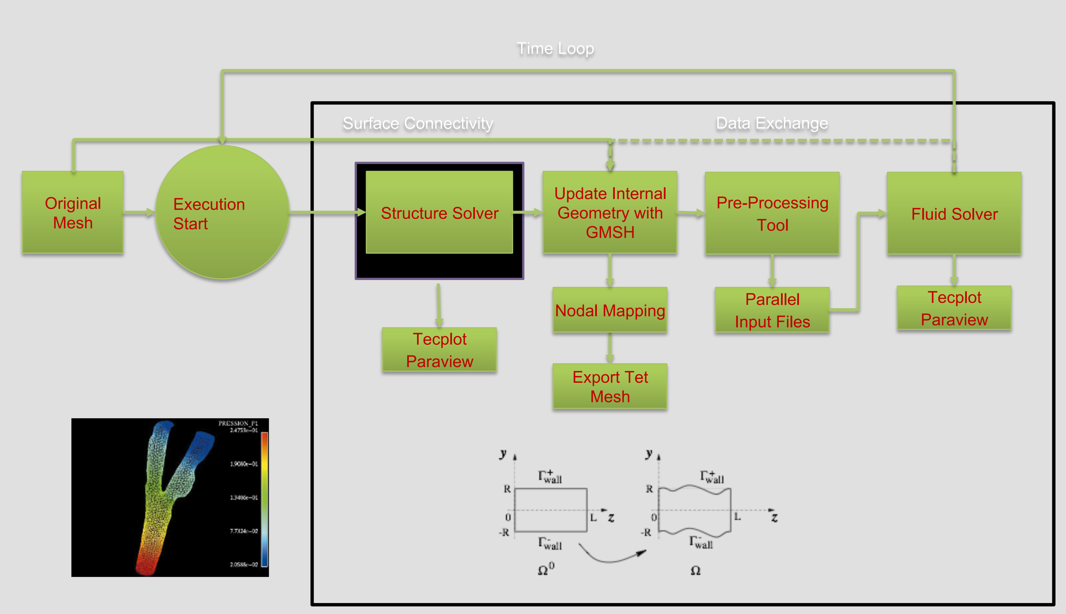
- Vessel Wall Equations(Structure Equations)

$$\begin{aligned} \nabla\cdot\tau^s - \nabla p^s &= 0, \\ \det(F) &= 1, \\ \tau^s &= G\left(F\cdot F^T - I\right), \\ F &= (\vec{\nabla}_0\vec{x})^T, \end{aligned}$$

1. Solve Navier-Stokes equations(INS) for blood flow velocity and pressure
2. Solve structure equations for radial and longitudinal deformations of the vessel wall
3. Update the mesh
4. Update radial velocity at vessel wall
5. $t = t + \Delta t$
6. Continue from Step 1

Parallel Computing, DIEL

- darter, or star1
- Parallel Interoperable Computational Mechanics Simulation System (PICMSS)
- Each responsible for several rows of grid
- 1cm diameter x 6cm length



Methodology

To solve INS:

Parallel Interoperable Computational Mechanics System Simulator(PICMSS) was chosen to solve INS.

- PICMSS

- parallel computational software
- solving equations with continuous Galerkin finite element method
- C program with MPI
- uses Trilinos iterative library for solving systems of linear equations generated internally by finite element method.
- 2D triangle and quadrilateral, and 3D tetrahedron and hexahedron master elements.
- fluid flow problems directly written in partial differential equation(PDE) template operator form

- Finite Element Method

This method divides the domain into parts and over each parts, uses some element functions to seek approximate solution then assembles the parts.

$$u(x,r) \approx u^e(x,r) = \sum_{j=1}^{n} U_j^e \psi_j^e(x,r)$$

$$\begin{aligned} \int \psi_i^e \nabla^2 u dV &= -\int \nabla\psi_i^e \cdot \nabla u dV + \oint \psi_i^e \cdot \nabla u ds \\ &= -\{U_j^e\}\int \nabla\psi_i^e\cdot\nabla\psi_j^e dV + \oint \psi_i^e \cdot \nabla u ds \end{aligned}$$

In PDE template operator form:

$$[\nabla^2](u) = -\{U_j^e\}\int \nabla\psi_i^e\cdot\nabla\psi_j^e dV = -[b2kk]\{U_j^e\}$$

X	X	X	X
X	X	X	X
X	X	X	X
X	X	X	X

To solve Structure Equations :

1. Use continuous Galerkin finite element method
2. Use Newmark method to solve system of second order PDE

-Newmark Method

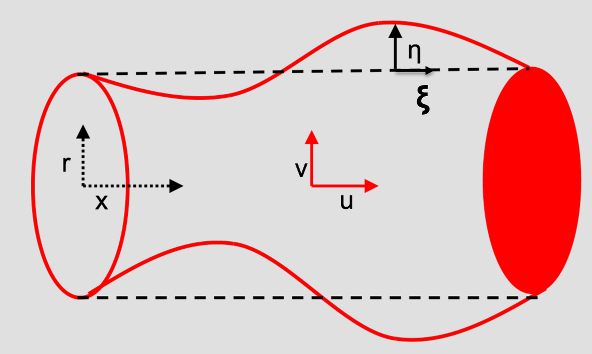
This method involves equations of the form:

$$[M]\{\frac{\partial^2\eta}{\partial t^2}\} + [C]\{\frac{\partial\eta}{\partial t}\} + [K]\{\eta\} = F$$

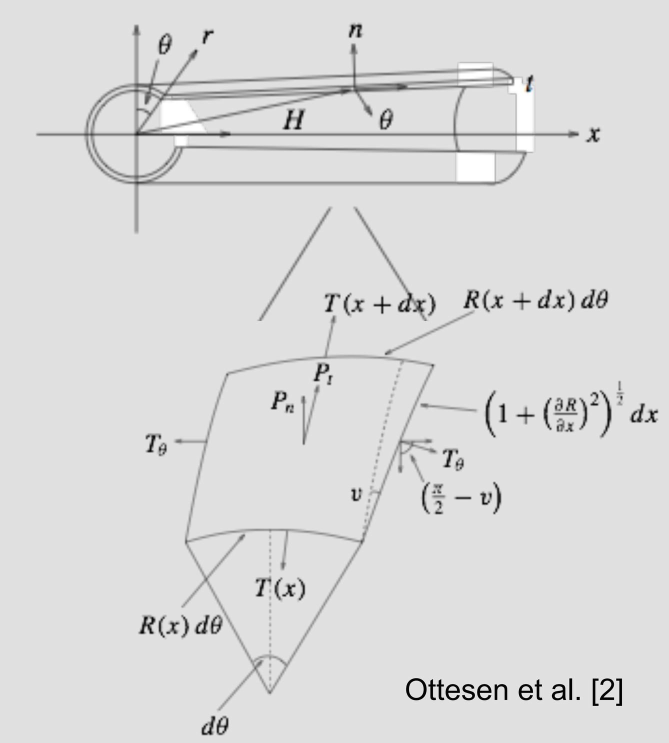
The solution of this equation for the Newmark Method is :

$$\begin{aligned} &([M] + \frac{\delta t}{2}[C] + \frac{\delta t^2}{4}[K])\{\frac{\partial^2\eta}{\partial t^2}\}_{n+1} \\ &= [F]_{n+1} - [C](\{\frac{\partial\eta}{\partial t}\}_n + \frac{\delta}{2}\{\frac{\partial^2\eta}{\partial t^2}\}_n) - [K](\{\eta\}_n + \delta t\{\frac{\partial\eta}{\partial t}\}_n + \frac{\delta t^2}{4}\{\frac{\partial^2\eta}{\partial t^2}\}_n) \\ &\{\eta\}_{n+1} = \{\eta\}_n + \delta t\{\frac{\partial\eta}{\partial t}\}|_n + \frac{\delta t^2}{4}(\{\frac{\partial^2\eta}{\partial t^2}\}_n + \{\frac{\partial^2\eta}{\partial t^2}\}_{n+1}) \\ &\{\frac{\partial\eta}{\partial t}\}_{n+1} = \{\frac{\partial\eta}{\partial t}\}_n + \frac{\delta t}{2}(\{\frac{\partial^2\eta}{\partial t^2}\}_n + \{\frac{\partial^2\eta}{\partial t^2}\}_{n+1}) \end{aligned}$$

1D structure and 2D axisymmetric Artery model



Quarteroni et al. [1]



Ottesen et al. [2]

- Axisymmetric INS

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial x} &= -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial x^2} - \frac{u}{r^2}) \\ \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial x} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial x^2}) \\ \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial x} &= 0, \end{aligned}$$

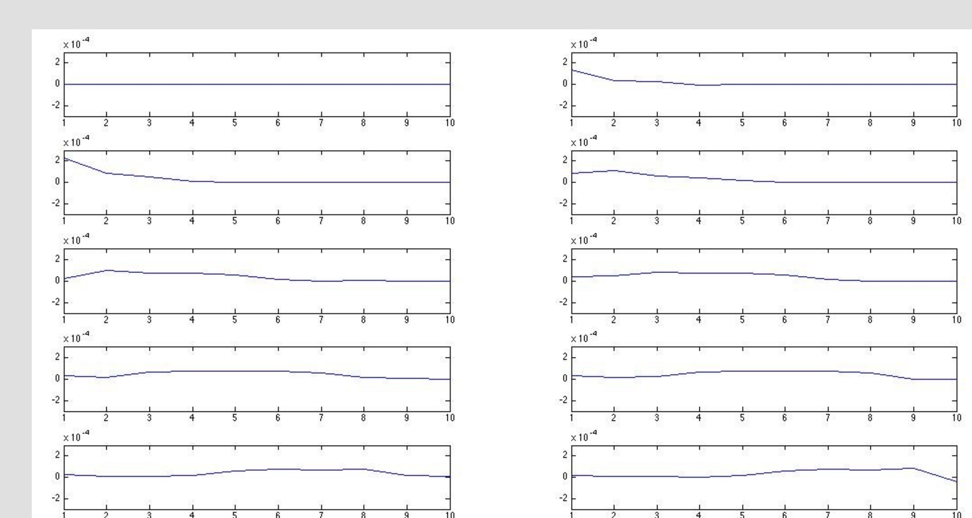
- 1D Structure Equations

$$\begin{aligned} &M_0\frac{\partial^2\xi}{\partial t^2} + L_x\frac{\partial\xi}{\partial t} + K_x\xi \\ &= \frac{E_x h}{1-\sigma_\theta\sigma_x}\frac{\partial^2\xi}{\partial x^2} + (\frac{T_{t_0} - T_{\theta_0}}{a} - \frac{E_x h\sigma_x}{a(1-\sigma_\theta\sigma_x)})\frac{\partial\eta}{\partial x} - \mu[\frac{\partial w}{\partial r} + \frac{\partial u}{\partial x}]_a \\ &M_0\frac{\partial^2\eta}{\partial t^2} + L_r\frac{\partial\eta}{\partial t} + K_r\eta \\ &= T_{t_0}\frac{\partial^2\eta}{\partial x^2} + (\frac{T_{\theta_0}}{a^2} - \frac{E_\theta h}{a^2(1-\sigma_\theta\sigma_x)})\eta + \frac{E_\theta h\sigma_\theta}{a(1-\sigma_\theta\sigma_x)}\frac{\partial\xi}{\partial x} + [p - 2\mu\frac{\partial u}{\partial r}]_a \end{aligned}$$

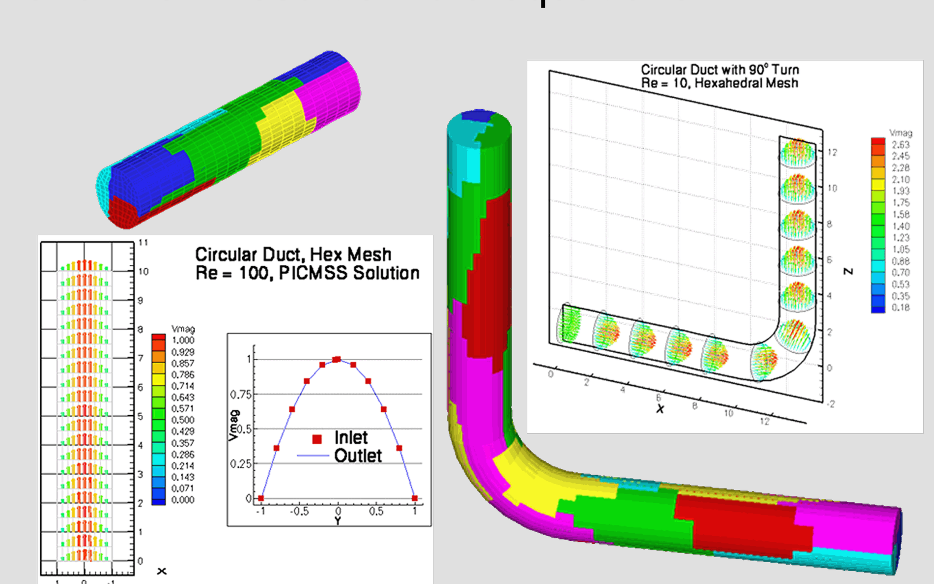
where E_i, $i = \theta, t$, is Young's modulus in the ith direction; h is the wall thickness; σ_i, $i = \theta, x$, is the Poisson ratio in the ith direction; and ϵ_i, $i = \theta, x$, is the displacement relative to the reference state;

Benchmark Results

- Benchmark result of 1D vessel wall



- Benchmark result of fluid equations



Future Work

- Run the code of 2D axisymmetric structure equations on PICMSS and compare with result of 1D serial code
- Solve full 3D fluid equations and structure equations
- Solve fully coupled fluid-structure equations
- Use DIEL to solve coupled equations

2D axisymmetric structure equations

- Simulate the vessel wall with no tangential velocity
- Use the same structure equations on 3D mesh but the differences are boundary conditions(red circle)

$$\begin{aligned} &M_0\frac{\partial^2\xi}{\partial t^2} + L_x\frac{\partial\xi}{\partial t} + K_x\xi \\ &= \frac{E_x h}{1-\sigma_\theta\sigma_x}\frac{\partial^2\xi}{\partial x^2} + (\frac{T_{t_0} - T_{\theta_0}}{a} - \frac{E_x h\sigma_x}{a(1-\sigma_\theta\sigma_x)})\frac{\partial\eta}{\partial x} - \mu[\frac{\partial w}{\partial r} + \frac{\partial u}{\partial x}]_a \\ &M_0\frac{\partial^2\eta}{\partial t^2} + L_r\frac{\partial\eta}{\partial t} + K_r\eta \\ &= T_{t_0}\frac{\partial^2\eta}{\partial x^2} + (\frac{T_{\theta_0}}{a^2} - \frac{E_\theta h}{a^2(1-\sigma_\theta\sigma_x)})\eta + \frac{E_\theta h\sigma_\theta}{a(1-\sigma_\theta\sigma_x)}\frac{\partial\xi}{\partial x} + [p - 2\mu\frac{\partial u}{\partial r}]_a \end{aligned}$$

Full 3D structure equations

D is the deformation matrix of vessel wall, and p is the pressure of the wall

$$\begin{aligned} &\frac{\partial}{\partial x_1}(F_{11}^2 + F_{12}^2 - 1 - p) + \frac{\partial}{\partial x_2}(F_{21}F_{11} + F_{22}F_{12}) = 0 \\ &\frac{\partial}{\partial x_1}(F_{21}F_{11} + F_{22}F_{12}) + \frac{\partial}{\partial x_2}(F_{21}^2 + F_{22}^2 - 1 - p) = 0 \\ &-\frac{\partial p}{\partial x_3} = 0 \\ &F_{11}F_{22} - F_{21}F_{12} = 0 \\ &F_{ij} = \frac{\partial D_i}{\partial x_j} \end{aligned}$$

References

[1] A. Quarteroni, M. Tuveri, A. Veneziani,``Computational vascular fluid dynamics: problems, models, and methods'',Comput Visual Sci, vol. 2, pp. 163-197, 2000.

[2] J. T. Ottesen, M. S. Olufsen, J. K. Larsen, Applied Mathematical Models in Human Physiology(Siam Monographs on Mathematical Modeling and Computation), SIAM, 2004.

Acknowledgements

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